4.5 linearizations, differentials

tangent line

Find the linearization to $y=\sqrt{x}$ at $x=4$

$y' = \frac{1}{2\sqrt{x}}$ $y'(4) = \frac{1}{4}$ tan line: $y = \frac{1}{4}(x-4)+2$

approximate $\sqrt{4.1}$

tan line $x=4.1$ $y=\frac{1}{4}(4.1-4)+2$

$\sqrt{4.1} = 2.02485$

$y = \frac{1}{4}(.1)+2$

$y = 2.025$

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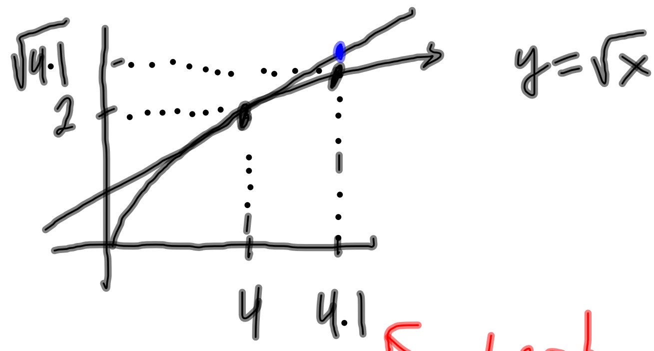
why did the tan line approximate f(x)



hard

close & easy

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approximate $\cos(1.75)$

1.75 – close to $\frac{\pi}{2} \approx 1.57\ldots$

easy, close

$y = \cos x$

tan line (linearization)

$y' = -\sin x$

$y = -1\left(x - \frac{\pi}{2}\right) + 0$

$y'\left(\frac{\pi}{2}\right) = -1$

$y(1.75) = -1\left(1.75 - \frac{\pi}{2}\right) \approx -.179$

$\cos(1.75) = -.178$

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$y \approx f'(x_0)(x - x_0) + y_0$

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approx $\sqrt[3]{8.3}$ $\qquad y = \sqrt[3]{x} = x^{1/3}$

$x_0 = 8 \quad y_0 = \sqrt[3]{8} = 2$ $\qquad y' = \frac{1}{3}x^{-2/3}$

close, easy $\qquad y'(8) = \frac{1}{3} 8^{-2/3}$

tan line $y = \frac{1}{12}(x - 8) + 2$ $\qquad = \frac{1}{3} \cdot 2^{-2} = \frac{1}{12}$

$\sqrt[3]{8.3} \approx \frac{1}{12}(8.3 - 8) + 2 = 2.025$

$\sqrt[3]{8.3} = 2.02469$

$x_0 = 8 \quad y_0 = 2$ $\qquad \Delta x = 8.3 - 8 = .3$

$x = 8.3 \quad y \approx 2.025$ $\qquad \Delta y \approx 2.025 - 2 = .025$

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$$\boxed{\begin{aligned} y(x) &\approx y'(x_0)(x-x_0) + y(x_0) \\ \text{or } & \; y'(x_0)\,\Delta x + y_0 \end{aligned}}$$

linear approximation

$$\boxed{\Delta y \approx y'(x_0)\,\Delta x}$$

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sphere: radius expands from $r=10$ to $r=10.2$

approximate the increase in volume

$V = \frac{4}{3}\pi r^3$ $\quad\Delta V \approx V'(r_0)\,\Delta r$

$r_0 = 10$ $\quad\Delta V \approx 4\pi \cdot 10^2 \cdot (.2) = 251.327$

$\Delta r = .2$ $\quad\Delta V = \frac{4}{3}\pi(10.2)^3 - \frac{4}{3}\pi(10^3) = 256.387$

$V'(r) = 4\pi r^2$

relative change in volume $\quad \frac{\Delta V}{V} \approx \frac{251.327}{\frac{4}{3}\pi 10^3} = .06$

percent change $= 6\%$

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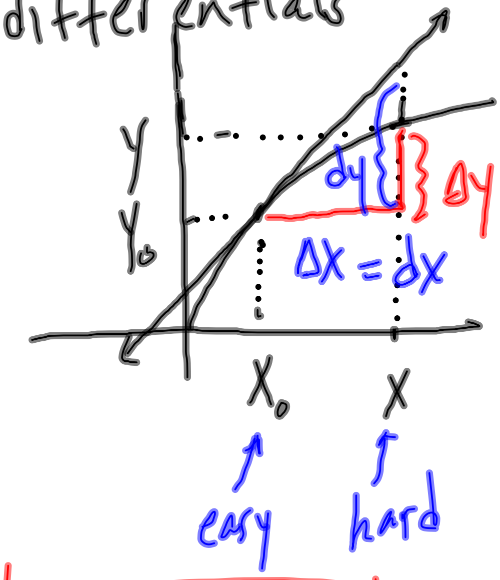
differentials



$$\boxed{\Delta y \approx dy = f'(x_0)\Delta x}$$

$\Delta x = dx$

$$\boxed{dy = \frac{dy}{dx} \cdot dx}$$

dx, dy are called differentials

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find $d(\tan(2x)) = 2\sec^2(2x) \cdot dx$

(differential of $\tan(2x)$)

easy $\rightarrow x = 0 \quad dx = .01 \qquad 2\sec^2(2\cdot 0)(.01)$

hard $x = 0 + .01$

$= .01$

$\Delta y \approx dy = .02$

approx $\tan(.02)$

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sphere - earth $r = 3959 \pm 0.1$ mi

$\uparrow$ dr

tolerance

surface area

$s = 4\pi r^2 \quad \frac{ds}{dr} = 8\pi r$

approx error (Δs)

$\Delta s \approx \frac{ds}{dr} \cdot dr = 8\pi r (0.1)$

$= 8\pi(3959)(0.1) = 9950$

EX 10

~~pct~~ % change $\frac{9950}{4\pi\, 3959^2} \times 100 =$

$.005\%$

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